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# Gauge-covariant vertex operators

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## Abstract

We derive Yang-Mills vertex operators for (super)string theory whose BRST invariance requires only the free gauge-covariant field equation and no gauge condition. Standard conformal field theory methods yield the three-point vertices directly in gauge-invariant form.

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# 1 Introduction

In string theory the usual vertex operators are not gauge invariant. For example, the condition  $k \cdot \epsilon = 0$  is imposed on the open-string vertex operator  $\epsilon \cdot \partial X e^{ik \cdot X}$  for the requirement of conformal weight 1 (see, e.g., [1]). On the other hand, nonlinear sigma models directly give gauge invariant results, but only order-by-order in  $\alpha'$ , and thus not the complete scattering amplitude [2].

In string theory there are two kinds of vertex operators in any amplitude – an integrated one  $\oint W$  and an unintegrated one  $V$ :

$$\mathcal{A} = \langle VVV \oint W \cdots \oint W \rangle \quad (1)$$

where for a gauge vector

$$W = A(X) \cdot \partial X \quad (2)$$

or in Fourier expansion

$$W = \epsilon \cdot \partial X e^{ik \cdot X} \quad (3)$$

Then, using the BRST operator “ $Q$ ”, we can find the unintegrated operator  $V$  [3]:

$$[Q, \oint W] = 0 \quad \Rightarrow \quad [Q, W] = \partial V \quad \Rightarrow \quad [Q, V] = 0 \quad (4)$$

In this paper, we will use the BRST operator and the integrated vertex operator of the massless vector to construct a gauge-covariant unintegrated vertex operator. This allows computation of amplitudes without fixing the gauge on external lines. Then the amplitude between 3 gauge bosons is computed and found to be the same as the 3-point YM vertex found from a gauge-invariant action. Also, the conformal symmetry of the amplitude is studied to show that the result follows from any 3-string vertex for string field theory [4]. This formalism is then generalized to the Neveu-Schwarz case.

## 2 Bosonic vertex

For the bosonic string, we calculate

$$Q = \oint \frac{1}{2\pi i} dz \left( -\frac{1}{4\alpha'} c \partial X \cdot \partial X + bc \partial c \right) \quad (5)$$

$$[Q, \epsilon \cdot \partial X e^{ik \cdot X}] = \oint \frac{1}{2\pi i} dz' \left( -\frac{1}{4\alpha'} c \partial' X \cdot \partial' X \right) \epsilon \cdot \partial X e^{ik \cdot X} \quad (6)$$

In the upper-half complex plane, the  $X$  propagator is  $-2\alpha' \ln|z' - z| \eta^{\mu\nu}$ , so

$$[Q, \epsilon \cdot \partial X e^{ik \cdot X}] = \partial(c\epsilon \cdot \partial X) e^{ik \cdot X} + c(\epsilon \cdot \partial X)(ik \cdot \partial X) e^{ik \cdot X} + \alpha'[(\epsilon \cdot \partial X)k^2 \partial c e^{ik \cdot X} - (ik \cdot \epsilon) \partial^2 c e^{ik \cdot X}] \quad (7)$$

The first two terms come from the two ways to contract a single pair of  $X^\mu(z')$  and  $X^\nu(z)$ , and the last two terms from the two ways to contract two pairs. To write it as a total derivative, using the *gauge-invariant* equation of motion of the free vector

$$\partial^\mu F_{\mu\nu} = 0 \quad \text{or} \quad k^2 \epsilon^\mu - k^\mu(k \cdot \epsilon) = 0 \quad (8)$$

we have

$$[Q, \epsilon \cdot \partial X e^{ik \cdot X}] = \partial[c\epsilon \cdot \partial X e^{ik \cdot X} - i\alpha'(\partial c)(\epsilon \cdot k) e^{ik \cdot X}] \quad (9)$$

Thus we get a BRST-invariant vertex operator for the gauge vector without gauge fixing:

$$V = c\epsilon \cdot \partial X e^{ik \cdot X} - i\alpha'(\partial c)(\epsilon \cdot k) e^{ik \cdot X} \quad (10)$$

$$\text{or} \quad V = cA \cdot \partial X - \alpha'(\partial c)(\partial \cdot A) \quad (11)$$

$\partial c$  is also the vertex operator for the Nakanishi-Lautrup field  $B$  [5]: In the gauge  $b_0 = 0$ ,  $B = \partial \cdot A$ , and the two  $\partial c$  terms cancel.

Under gauge transformations,

$$\delta A^\mu = \partial^\mu \lambda \quad \Rightarrow \quad \delta V = c(\partial^\mu \lambda)(\partial X_\mu) - \alpha'(\partial c)\partial_\mu(\partial^\mu \lambda) \quad (12)$$

which can be written as the commutator between  $Q$  and  $\lambda$

$$\begin{aligned} [Q, \lambda] &= \oint \frac{1}{2\pi i} dz' [-\frac{1}{4\alpha'} c \partial' X(z') \cdot \partial' X(z')] \lambda[X(z)] \\ &= c(\partial^\mu \lambda)(\partial X_\mu) - \alpha'(\partial c)\partial_\mu(\partial^\mu \lambda) \\ &= \delta V \end{aligned} \quad (13)$$

It is easy to see the integrated operator is gauge invariant

$$\delta \oint W = \oint \partial X^\mu \partial_\mu \lambda = \oint \partial \lambda = 0 \quad (14)$$

so matrix elements are also:

$$\delta_{\lambda_1} \mathcal{A}_n = \langle \delta V_1 V_2 V_3 \oint W \cdots \oint W \rangle = \langle [Q, \lambda_1] V_2 V_3 \oint W \cdots \oint W \rangle = 0 \quad (15)$$

where the vacuum is BRST invariant, and similarly for the gauge transformations of  $V_2$  and  $V_3$ .

### 3 Bosonic three-point amplitudes

The simplest case is the 3-point amplitude:

$$\begin{aligned}\mathcal{A}_3 &= -\frac{ig_{YM}}{2\alpha'} \langle V_1 V_2 V_3 \rangle \\ &= -\frac{ig_{YM}}{2\alpha'} \langle \prod_{i=1}^3 \underbrace{[c(y_i) \epsilon_i \cdot \partial X(y_i) e^{ik_i \cdot X(y_i)}]_{G(y_i)}}_{G(y_i)} - \underbrace{i\alpha' \partial c(y_i) (\epsilon_i \cdot k_i) e^{ik_i \cdot X(y_i)}}_{H(y_i)} \rangle\end{aligned}\quad (16)$$

Considering the lowest order of  $\alpha'$ , only 2 terms contribute:

$$\langle \prod_{i=1}^3 G(y_i) \rangle \quad \text{and} \quad \langle G(y_i) \prod_{i \neq j} H(y_j) \rangle \quad (17)$$

Using

$$\begin{aligned}\langle c(y_1) c(y_2) c(y_3) \rangle &= y_{12} y_{13} y_{23} \\ \langle \partial_{y_1} c(y_1) c(y_2) c(y_3) \rangle &= \partial_{y_1} (y_{12} y_{13} y_{23}), \dots\end{aligned}\quad (18)$$

and the propagator between  $X^\mu(z')$  and  $X^\nu(z)$ , and contracting 2 of the  $\partial X$ 's with each other, the amplitude between 3 gauge bosons is:

$$\begin{aligned}\mathcal{A}_3^{(1)} &= ig_{YM} (2\pi)^D \delta^D(\Sigma_i k_i) [(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot k_{12}) + (\epsilon_2 \cdot \epsilon_3)(\epsilon_1 \cdot k_{23}) + (\epsilon_3 \cdot \epsilon_1)(\epsilon_2 \cdot k_{31})] \\ &\quad \times |y_{12}|^{2\alpha' k_1 \cdot k_2} |y_{13}|^{2\alpha' k_1 \cdot k_3} |y_{23}|^{2\alpha' k_2 \cdot k_3}\end{aligned}\quad (19)$$

where  $k_{ij} = k_i - k_j$ . To the lowest order in  $\alpha'$ , the last factor in  $\mathcal{A}_3$ ,

$$|y_{12}|^{2\alpha' k_1 \cdot k_2} |y_{13}|^{2\alpha' k_1 \cdot k_3} |y_{23}|^{2\alpha' k_2 \cdot k_3} \rightarrow 1 \quad (20)$$

when  $\alpha' \rightarrow 0$ . This amplitude corresponds to the cubic part of

$$\frac{1}{g_{YM}^2} \int d^{26}x \left[ -\frac{1}{4} \text{Tr}(F^{\mu\nu} F_{\mu\nu}) \right] \quad (21)$$

in Yang-Mills theory, without gauge fixing (or use of a gauge condition).

With the same technology but tedious calculation, the 3-point amplitude to second order in  $\alpha'$  contributed to by

$$\langle G(y_1) G(y_2) G(y_3) \rangle, \quad \langle G(y_1) G(y_2) H(y_3) \rangle, \dots, \quad \text{and} \quad \langle G(y_1) H(y_2) H(y_3) \rangle, \dots \quad (22)$$

where  $\dots$  represents the permutation of  $y_i$ 's, is:

$$\begin{aligned}\mathcal{A}_3^{(2)} &= 2i\alpha' g_{YM} (2\pi)^{26} \delta^{26} \left( \sum_i k_i \right) \times \\ &\quad [(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_3)(\epsilon_3 \cdot k_1) - (\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_1)(\epsilon_3 \cdot k_2)]\end{aligned}\quad (23)$$

The  $H^3$  contribution vanishes.

The remaining contributions to the amplitude, from expanding the factors

$$|y_{12}|^{2\alpha' k_1 \cdot k_2} |y_{13}|^{2\alpha' k_1 \cdot k_3} |y_{23}|^{2\alpha' k_2 \cdot k_3} \quad (24)$$

are just zero because

$$k_1 \cdot k_2 = \frac{1}{2}(k_3^2 - k_1^2 - k_2^2), \dots \quad \text{and} \\ k_1^2 [(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot k_{12}) + (\epsilon_2 \cdot \epsilon_3)(\epsilon_1 \cdot k_{23}) + (\epsilon_3 \cdot \epsilon_1)(\epsilon_2 \cdot k_{31})] = 0, \dots \quad (25)$$

by using only the *gauge-covariant* equation of motion (8) and momentum conservation  $k_1 + k_2 + k_3 = 0$ .

This  $\mathcal{A}_3^{(2)}$  gives exactly the cubic term in the YM field:

$$\frac{-2i\alpha'}{3g_{YM}^2} \text{Tr}(F_\mu{}^\nu F_\nu{}^\omega F_\omega{}^\mu) \quad (26)$$

Another simple 3-point amplitude we can calculate with this gauge independent vertex operator is between 1 gauge boson and 2 tachyons. Using  $V_t = ce^{ik \cdot X}$ , to lowest order in  $\alpha'$ ,

$$-ig_{YM} g_0^2 \langle V(y_1) V_t(y_2) V_t(y_3) \rangle = -ig_{YM} \epsilon_1 \cdot (k_2 - k_3) (2\pi)^{26} \delta^{26}(\Sigma_i k_i) \quad (27)$$

with  $g_0 = (2\alpha')^{1/2} g_{YM}$  is the coupling constant for tachyons. This corresponds to

$$\frac{1}{g_{YM}^2} \left[ -\frac{1}{2} \text{Tr}(D_\mu \phi D^\mu \phi) \right] \quad \text{with} \quad D_\mu \phi = \partial \phi - i[A_\mu, \phi] \quad (28)$$

in quantum field theory.

One thing we can observe is that the amplitude is independent of the conformal map to the complex plane. This can be verified by checking the conformal transformation of the vertex operator:

$$\delta V = \oint \lambda(z') T(z') V(z) \quad (29)$$

The world-sheet energy momentum tensor includes two parts,

$$T(z') = T^m(z') + T^g(z') \\ T^m(z) = -\frac{1}{4\alpha'} \partial X_\mu \partial X^\mu \\ T^g(z) = (\partial b)c - 2\partial(bc) \quad (30)$$

Using the operator products of ghost  $c$  and antighost  $b$ ,

$$b(z_1)c(z_2) \sim \frac{1}{z_{12}} \quad (31)$$

and the equation of motion (8),

$$\delta V = \lambda \partial V + \alpha' k^2 (\partial \lambda) V \quad (32)$$

So the finite transformation of the operator is

$$V'(z') = \left( \frac{dz'}{dz} \right)^{-\alpha' k^2} V(z) \quad (33)$$

In string field theory, an arbitrary 3-point vertex can be defined by

$$\langle V[h_1(0)]V[h_2(0)]V[h_3(0)] \rangle = \langle h_1[V(0)]h_2[V(0)]h_3[V(0)] \rangle \quad (34)$$

with  $h_i$  arbitrary maps of  $z = 0$  into the upper complex plane [4]. This differs from the previous expression only by terms with extra factors of  $k_i^2$ . By the previous argument (25), such terms vanish by the gauge-invariant equation of motion.

## 4 Neveu-Schwarz vertex

This formalism can also be generalized to the NSR string: Start from the integrated vertex operator  $\oint \epsilon \cdot D_\theta X e^{ik \cdot X(Z)}$  with  $Z = (z, \theta)$ ,  $X^\mu(Z) = x^\mu(z) + i\theta\psi(z)$  and  $D \equiv D_\theta = \partial_\theta + \theta\partial_z$ . Here we will use the language for the Neveu-Schwarz string of the “Big Picture” [3]. Then as in the bosonic case, if the commutator can be written as  $[Q, W] = D_\theta V$ , then  $[Q, V] = 0$ . To simplify the calculation, we replace  $X$  by  $(\alpha'/2)^{1/2} X$ . Then

$$W = (\alpha'/2)^{1/2} \epsilon \cdot D_\theta X e^{ik \cdot (\alpha'/2)^{1/2} X(Z)} \quad (35)$$

and the propagator

$$X^\mu(z', \theta') X^\nu(z, \theta) \sim -4 \ln |z' - z - \theta' \theta| \eta^{\mu\nu} \quad (36)$$

We also write  $\sqrt{\alpha'/2} \epsilon$  as  $\epsilon$  and  $\sqrt{\alpha'/2} k$  as  $k$  and restore these factors in the final result.

The BRST operator  $Q$  in the NSR string is

$$Q = \frac{1}{2\pi i} \oint dz' d\theta' (CT^x + \frac{1}{2} CT^g) \quad (37)$$

with

$$T^x = -\frac{1}{8} C(z', \theta') (D_{\theta'} X^\mu) (\partial' X_\mu) \quad (38)$$

and

$$T^g = \frac{1}{2} (D_{\theta'} B) (D_{\theta'} C) - \frac{3}{2} B (\partial' C) - (\partial' B) C \quad (39)$$

Here  $C = c + \theta\gamma$  and  $B = \beta + \theta b$ , where  $c$  and  $b$  are anticommuting superconformal ghost and antighost, and  $\beta$  and  $\gamma$  are commuting superconformal ghost and antighost.

Because  $W$  contains no ghosts, it is only necessary to compute

$$[Q, W] = \frac{1}{2\pi i} \oint dz' d\theta' (-\frac{1}{8}) [C(z', \theta') (D_{\theta'} X^\mu) (\partial' X_\mu)] [\epsilon \cdot D_\theta X e^{ik \cdot X(Z)}] \quad (40)$$

Using

$$\oint dz' d\theta' \frac{1}{z' - z - \theta'\theta} [f(z') + \theta' g(z')] = D_\theta [f(z) + \theta g(z)] \quad (41)$$

and

$$\oint dz' d\theta' \frac{\theta' - \theta}{z' - z} [f(z') + \theta' g(z')] = f(z) + \theta g(z) \quad (42)$$

the result for  $V$  is

$$\begin{aligned} V &= -D_\theta [C(\epsilon \cdot D_\theta X) e^{ik \cdot X(Z)}] + \frac{1}{2} (D_\theta C) (D_\theta X \cdot \epsilon) e^{ik \cdot X(Z)} \\ &\quad - 2i(\epsilon \cdot k) (\partial C) e^{ik \cdot X(Z)} \\ &= G + H \end{aligned} \quad (43)$$

where  $G$  represents the first two terms and  $H$  represents the last term.

To see the gauge invariance of the amplitude, we should know the gauge transformation of the vertex operator  $\delta V$ . Writing the vertex operator as:

$$V = -D_\theta [C A^\mu (D_\theta X_\mu)] - 2(\partial C) (\partial_\mu A^\mu) + \frac{1}{2} (D_\theta C) A^\mu (D_\theta X_\mu), \quad (44)$$

then the gauge transformation of the vertex operator is

$$\begin{aligned} \delta V &= -D_\theta [C(\partial^\mu \lambda) (D_\theta X_\mu)] - 2(\partial C) [\partial_\mu (\partial^\mu \lambda)] + \frac{1}{2} (D_\theta C) (D_\theta X_\mu) (\partial^\mu \lambda) \\ &= -\frac{1}{2} (D_\theta C) (D_\theta X_\mu) \partial^\mu \lambda - 2(\partial C) \square \lambda + C(\partial X_\mu) \partial^\mu \lambda \\ &\quad - C(D_\theta X_\mu) (D_\theta X_\nu) \partial^\nu \partial^\mu \lambda \end{aligned} \quad (45)$$

under  $A \rightarrow A + \partial_\mu \lambda$ . The last term is zero because  $D_\theta X_\mu$  and  $D_\theta X_\nu$  anticommute.  $\delta V$  can be written as the commutator of the BRST operator and the function  $\lambda$ :

$$\begin{aligned} [Q, \lambda(X(z, \theta))] &= \frac{1}{2\pi i} \oint dz' d\theta' \frac{1}{8} C(z', \theta') (D_{\theta'} X^\mu) (\partial' X_\mu) \lambda(X(z, \theta)) \\ &= -\frac{1}{2} D_\theta (C D_\theta X_\mu) \partial^\mu \lambda - 2(\partial C) \square \lambda + \frac{1}{2} C(\partial X_\mu) \partial^\mu \lambda \end{aligned} \quad (46)$$

So the amplitude  $\langle VVV \oint W \cdots \oint W \rangle$  is gauge invariant just as in the bosonic case.

## 5 Neveu-Schwarz three-point amplitude

Similarly to the bosonic case, we are going to use this vertex operator in the NSR string to compute the 3-point amplitude between 3 YM gauge bosons. First of all, we should know the correlation function

$$\begin{aligned} & \langle 0 | C(z_1, \theta_1) C(z_2, \theta_2) C(z_3, \theta_3) | 0 \rangle \\ &= \theta_1 \theta_2 z_3 (z_1 + z_2) + \theta_2 \theta_3 z_1 (z_2 + z_3) + \theta_3 \theta_1 z_2 (z_3 + z_1) \end{aligned}$$

Then using the propagator between  $X$ 's as (36), the anticommutation relation between  $C$ 's and  $D_\theta$ 's, and the gauge-covariant equation of motion (8), after a more-complicated calculation, to the lowest order in  $\alpha'$ , we find

$$\begin{aligned} \mathcal{A}_3 &= \frac{2g_{YM}}{\alpha'^2} \langle V(z_1, \theta_1) V(z_2, \theta_2) V(z_3, \theta_3) \rangle \\ &= ig_{YM} (2\pi)^D \delta^D(\Sigma_i k_i) [(\epsilon_1 \cdot \epsilon_2)(\epsilon_3 \cdot k_{12}) + (\epsilon_2 \cdot \epsilon_3)(\epsilon_1 \cdot k_{23}) + (\epsilon_3 \cdot \epsilon_1)(\epsilon_2 \cdot k_{31})] \end{aligned} \quad (47)$$

Again this amplitude corresponds to the  $Tr(F^{\mu\nu} F_{\mu\nu})$  term in the YM theory.

This amplitude is independent of the anticommuting coordinate  $\theta$ , as expected. It is also independent of  $z$ , as in the bosonic case. We will discuss this more later. One thing different from the bosonic case is that  $F^3$  terms cannot be supersymmetrized. This corresponds to the absence of  $k^3$  terms in the NSR string amplitude. To verify this, it is necessary to compute to the second order in  $\alpha'$  in the amplitude.

To second order in  $\alpha'$ , factors like

$$|y_{ij} - \theta_i \theta_j|^{2\alpha' k_i \cdot k_j}, \quad D_{\theta_k} |y_{ij} - \theta_i \theta_j|^{2\alpha' k_i \cdot k_j}, \quad D_{\theta_k} D_{\theta_l} |y_{ij} - \theta_i \theta_j|^{2\alpha' k_i \cdot k_j}, \dots \quad (48)$$

are involved in the amplitude  $\mathcal{A}_3$ . To lowest order in  $\alpha'$ , the first factor is just one, the rest zero. But the expansion of  $2\alpha' k_i \cdot k_j$  contributes to the second order in  $\alpha'$ , as do

$$\begin{aligned} & \langle G(y_1) G(y_2) G(y_3) \rangle, \\ & \langle G(y_1) G(y_2) H(y_3) \rangle, \dots, \\ & \langle G(y_1) H(y_2) H(y_3) \rangle, \dots \end{aligned} \quad (49)$$

The term from

$$\langle H(y_1) H(y_2) H(y_3) \rangle \quad (50)$$

vanishes.



We calculate the expansion of  $|y_{ij} - \theta_i \theta_j|^{2\alpha' k_i \cdot k_j}$  and its derivatives  $D_{\theta_k} |y_{ij} - \theta_i \theta_j|^{2\alpha' k_i \cdot k_j}$ , etc., using

$$\begin{aligned}
& |y_{12} - \theta_1 \theta_2|^a |y_{23} - \theta_2 \theta_3|^b |y_{31} - \theta_3 \theta_1|^c \\
&= [1 - \frac{a}{y_{12}} \theta_1 \theta_2 - \frac{b}{y_{23}} \theta_2 \theta_3 - \frac{c}{y_{31}} \theta_3 \theta_1] |y_{12}|^a |y_{23}|^b |y_{31}|^c \\
&= [D_{\theta_1}(|y_{12}|^a \theta_{12})][D_{\theta_2}(|y_{23}|^b \theta_{23})][D_{\theta_3}(|y_{31}|^c \theta_{31})]
\end{aligned} \tag{51}$$

to the first order in  $a, b, c$ . Then

$$\begin{aligned}
& D_{\theta_1}(|y_{12} - \theta_1 \theta_2|^a |y_{23} - \theta_2 \theta_3|^b |y_{31} - \theta_3 \theta_1|^c) \\
&= (\partial_1 |y_{12}|^a) |y_{23}|^b |y_{31}|^c \theta_{12} + |y_{12}|^a |y_{23}|^b (\partial_3 |y_{31}|^c) \theta_{31} \\
&\quad + \theta_1 \theta_2 \theta_3 [(\partial_1 |y_{12}|^a) \partial_3 (|y_{23}|^b |y_{31}|^c) + \partial_2 (|y_{12}|^a |y_{23}|^b) (\partial_3 |y_{31}|^c)]
\end{aligned} \tag{52}$$

$$D_{\theta_1} D_{\theta_2} (|y_{12} - \theta_1 \theta_2|^a |y_{23} - \theta_2 \theta_3|^b |y_{31} - \theta_3 \theta_1|^c) = \frac{a}{y_{12}} + \frac{a}{y_{12}^2} \theta_1 \theta_2, \dots \tag{53}$$

The detailed calculation is too long to show here. But combining both contributions, the 3-point amplitude to the second order in  $\alpha'$  is just zero as predicted.

To study the vertex position dependence of the amplitude, we compute the conformal transformation of  $V$

$$\delta V = \frac{1}{2\pi i} \oint dz' d\theta' \lambda(T^x + T^g)(z', \theta') V(z, \theta) \tag{54}$$

with  $T^x$  and  $T^g$  defined in eqs. (38-39). Using the OPE of  $B$  and  $C$

$$B(z_1, \theta_1) C(z_2, \theta_2) \sim \frac{\theta_1 - \theta_2}{z_1 - z_2}, \tag{55}$$

$$\delta V = (2k^2)(\partial\lambda)V + \frac{1}{2}(D_\theta)(D_\theta V) + \lambda(\partial V) \tag{56}$$

Replacing  $k$  by  $\sqrt{\alpha'/2}k$  as mentioned in the beginning of section 4, and taking the infinitesimal superconformal transformation  $\lambda = 2\theta\eta$  with  $\eta(z)$  anticommuting,

$$\delta V = (2\alpha' k^2)(\theta\partial\eta)V + \eta Q_\theta V \tag{57}$$

where  $Q_\theta = \partial_\theta - \theta\partial_z$ .

Thus the vertex operator has the weight  $\alpha' k^2$  and transforms as

$$V'(z', \theta') = (D_\theta \theta')^{-2\alpha' k^2} V(z, \theta) \tag{58}$$

So the amplitude transforms as

$$\begin{aligned}
& \langle V'(z'_1, \theta'_1) V'(z'_2, \theta'_2) V'(z'_3, \theta'_3) \rangle \\
&= (D_{\theta_1} \theta'_1)^{-2\alpha' k_1^2} (D_{\theta_2} \theta'_2)^{-2\alpha' k_2^2} (D_{\theta_3} \theta'_3)^{-2\alpha' k_3^2} \langle V(z_1, \theta_1) V(z_2, \theta_2) V(z_3, \theta_3) \rangle
\end{aligned} \tag{59}$$

The expansion of  $\alpha' k_i^2$  gives just one, using  $\square F = 0$ . This implies

$$\langle V'(z'_1, \theta'_1) V'(z'_2, \theta'_2) V'(z'_3, \theta'_3) \rangle = \langle V(z_1, \theta_1) V(z_2, \theta_2) V(z_3, \theta_3) \rangle \tag{60}$$

and no higher order terms in  $\alpha'$ .

## 6 Conclusions

We have given a general construction for gauge-covariant vertex operators, and applied it to the YM vertex in the string and superstring in 3-point amplitudes. This method allows direct calculation of gauge-invariant results, analogous to nonlinear sigma models, and can also be applied to string field theory. Possible future applications include higher-point amplitudes.

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